

# 1.5 The pseudo-F statistic

Once the partitioning has been done we are ready to calculate a test statistic associated with the general multivariate null hypothesis of no differences among the groups. For this, following R. A. Fisher's lead, a pseudo- $F$  ratio is defined as:

$$F = \frac{SS_A / (a - 1)}{SS_{Res} / (N - a)} \tag{1.3}$$

where  $(a - 1)$  are the degrees of freedom associated with the factor and  $(N - a)$  are the residual degrees of freedom. It is clear here that, as the pseudo- $F$  statistic in (1.3) gets *larger*, the likelihood of the null hypothesis being true *diminishes*. Interestingly, if there is only one variable in the analysis and one has chosen to use Euclidean distance, then the resulting PERMANOVA  $F$  ratio is exactly the same as the original  $F$  statistic in traditional ANOVA<sup>10</sup> ( [Fisher \(1924\)](#) ). In general, however, the PERMANOVA  $F$  ratio should be thought of as a “pseudo”  $F$  statistic, because it does *not* have a known distribution under a true null hypothesis. There is only one situation for which this distribution is known and corresponds to Fisher's traditional  $F$  distribution, namely: (i) if the analysis is being done on a single response variable *and* (ii) the distance measure used was Euclidean distance *and* (iii) the single response variable is normally distributed. In all other cases (multiple variables, non-normal variables and/or non-Euclidean dissimilarities), all bets are off! Therefore, in general, we cannot rely on traditional tables of the  $F$  distribution to obtain a  $P$ -value for a given multivariate data set.

Some other test statistics based on resemblance measures (and using randomization or permutation methods to obtain  $P$ -values, see the next section) have been suggested for analysing one-way ANOVA designs (e.g., such as the average between-group similarity divided by the average within-group similarity as outlined by [Good \(1982\)](#) and [Smith, Pontasch & Cairns \(1990\)](#) , see also all of the good ideas in the book by [Mielke & Berry \(2001\)](#) and references therein). Unlike pseudo- $F$ , however, these can be limited in that they may not necessarily yield straightforward extensions to multi-way designs.

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<sup>10</sup> In fact, a nice way to familiarise oneself with the routine is to do a traditional univariate ANOVA using some other package and compare this with the outcome from the analysis of that same variable based on Euclidean distances using PERMANOVA.

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