

# A3 Index to mathematical notation and symbols

## Matrices and vectors

**A** = matrix containing elements  $a_{ij} = \frac{1}{2} d_{ij}^2$

**B** = matrix of variables ( $N \times s$ ) that are linear combinations of normalised **X** variables having maximum correlation with CAP axes

**C** = matrix of CAP axes ( $N \times s$ ), standardised by the square root of their respective eigenvalues

**D** = matrix containing elements  $d_{ij}$  corresponding to distances or dissimilarities

**G** = Gower's centred matrix, consisting of elements  $g_{ij} = a_{ij} - \bar{a}_{i.} - \bar{a}_{.j} + \bar{a}_{..}$

**H** = 'hat' matrix =  $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ , used as a projection matrix for regression models

**I** = identity matrix, with 1's along the diagonal and 0's elsewhere

**Q** = matrix of PCO axes, standardised by the square root of their respective eigenvalues

$\mathbf{Q}^0$  = matrix of PCO axes, orthonormalised to SSCP = **I** ('sphericised')

**U** = matrix whose columns contain the left singular vectors from a singular value decomposition (SVD) of a matrix (e.g.,  $\mathbf{X} = \mathbf{U}\mathbf{W}\mathbf{V}'$ ); if **X** is ( $N \times q$ ) and  $q < N$ , then **U** is ( $N \times q$ )

**V** = matrix whose columns contain the right singular vectors from a singular value decomposition (SVD) of a matrix (e.g.,  $\mathbf{X} = \mathbf{U}\mathbf{W}\mathbf{V}'$ ); if **X** is ( $N \times q$ ) and  $q < N$ , then **V** is ( $q \times N$ )

**W** = diagonal matrix of eigenvalues from a singular value decomposition (SVD) of a matrix (e.g.,  $\mathbf{X} = \mathbf{U}\mathbf{W}\mathbf{V}'$ ); if **X** is ( $N \times q$ ) and  $q < N$ , then **W** is ( $q \times q$ )

**X** = matrix of predictor variables ( $N \times q$ ) (often a set of environmental variables)

$\mathbf{X}^0$  = matrix of **X** variables, orthonormalised to SSCP = **I** ('sphericised')

**Y** = matrix of response variables ( $N \times p$ ) (often a set of species variables)

$\mathbf{Y}^0$  = matrix of **Y** variables, orthonormalised to SSCP = **I** ('sphericised')

$\hat{\mathbf{Y}}$  =  $\mathbf{H}\mathbf{Y}$  = matrix of fitted values ( $N \times p$ )

$\mathbf{y}_{ij}$  = vector of  $p$  response variables for the  $j$ th observation in the  $i$ th group

$\bar{\mathbf{y}}_i$  = the centroid vector of  $p$  response variables for group  $i$

**Z** = matrix of dbRDA canonical axes ( $N \times s$ )

## Letters

$a, b, c, \text{etc.}$  = number of levels of factor A, B, C, etc... in an ANOVA experimental design

$AIC$  = multivariate analogue to Akaike's 'An information criterion'

$AIC_c$  = multivariate analogue to the small-sample-size corrected version of  $AIC$

$B_{iI}$  = the  $i$ th variable in the space of normalised **X** variables that has maximum correlation with the  $I$ th coordinate axis ( $C_{iI}$ ) from a CAP analysis

$BIC$  = multivariate analogue to Schwarz's 'Bayesian information criterion'

$C_{l}$  = the  $l$ th coordinate axis scores from a CAP analysis

$d_{ij}$  = distance or dissimilarity between sample  $i$  and sample  $j$

$df$  = degrees of freedom

$F$  = pseudo- $F$  statistic for testing hypotheses in PERMANOVA or DISTLM

$i$  = index used for samples (i.e.,  $i = 1, \dots, N$ ) or index used for groups ( $i = 1, \dots, a$ )

$j$  = second index used for samples (i.e.,  $j = 1, \dots, N$ ) **or** index used for replicates within a group ( $j = 1, \dots, n$ )

$k$  = index used for variables (i.e.,  $k = 1, \dots, p$  or else  $k = 1, \dots, q$ )

$l$  = index used for canonical axes or eigenvalues for either dbRDA **or** CAP (i.e.,  $l = 1, \dots, s$ ) **or** either the abbreviation for 'log-likelihood' or the 'length' of a vector (depending on context).

$m$  = number of PCO axes chosen as a subset for analysis by CAP

$MC$  = Monte Carlo

$MS$  = mean square

$N$  = total number of samples

$n$  = number of samples (replicates) within a group or cell in an experimental design

$P$  =  $P$ -value associated with the test of a null hypothesis

$p$  = number of multivariate response variables in matrix  $\mathbf{Y}$

$q$  = total number of predictor variables in matrix  $\mathbf{X}$

$r$  = Pearson correlation coefficient

$R$  = the ANOSIM  $R$  statistic (see [Clarke \(1993\)](#) )

$R^2$  = proportion of explained variation from a model

$s$  = number of canonical eigenvalues and associated canonical axes obtained from either a dbRDA **or** a CAP analysis  $SS$  = sum of squares

$SSCP$  = sum of squares and cross products

$SVD$  = singular value decomposition

$t$  = pseudo- $t$  statistic =  $\sqrt{\text{pseudo-}F}$

$tr$  = 'trace' of a matrix = the sum of the diagonal elements

$X_k$  = the  $k$ th predictor variable

$Y_k$  = the  $k$ th response variable

$z_{ij}$  = distance to group centroid for the  $j$ th replicate within the  $i$ th group.

## Greek symbols and matrices

$\alpha$  = significance level chosen for a test (usually  $\alpha = 0.05$ ).

$\delta_l^2$  = the  $l$ th eigenvalue from a CAP analysis, a squared canonical correlation

$\Delta$  = diagonal matrix containing the square roots of the eigenvalues from a CAP analysis (a capital delta)

$\gamma_l^2$  = the  $l$ th eigenvalue from a dbRDA analysis, a portion of the explained (regression) sum of squares from a dbRDA model.

$\Gamma$  = diagonal matrix containing the square roots of the eigenvalues from a dbRDA analysis (a capital gamma)

$\lambda_i$  = the  $i$ th eigenvalue from a PCO analysis

$\Lambda$  = diagonal matrix of eigenvalues from a PCO analysis (a capital lambda)

$\nu$  = number of parameters in a particular model during model selection

$\rho$  = Spearman rank correlation ( $\rho$ )

$\sum$  = sum over the relevant index

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